Abstract - The process of delivering ready-mix concrete (RMC) to construction projects is a critical supply chain activity. This study models decision aiding method to assess RMC unloading type involving multiple stakeholders and evaluation criteria. The uncertainty of criteria weights set by expert judgment can be simulated in random ways within a prioritization matrix. The ranking is performed by grey relational grade systems based on individual preference. Using the illustrative example from an engineering corporation, the case study demonstrates the practicability of the proposed method.

Keywords - Supply chain management; Decision analysis and methods; Project management; Ready-mixed concrete.

I. INTRODUCTION

The process of delivering ready-mix concrete (RMC) to construction projects is a critical supply chain management activity. The process depends on the concrete truck mixer, the concrete mobile pump, and the project site environment. However, decision aids (DAs) for evaluating RMC supply chain processes need further development, especially for the RMC unloading type. Particularly, selecting the most appropriate RMC unloading type by considering trade-offs among multiple stakeholders and criteria is actually a highly complex multi-expert multi-criteria decision aid (ME-MCDA) problem.

In practice, multi-expert/group conflict and uncertainty may also occur when decision makers (DMs) apply their personally preferred structure in the alternative evaluation. The comprehension, analysis and support of the process are even more difficult when deciding how the problem should be handled and what decision should be made. Thus, this study proposes novel supporting tools for integrating DA strategies to produce robust, reliable, and objective decision making results. The proposed decision aid model helps DMs obtain the best solutions to diverse ME-MCDA problems.

II. LITERATURE SURVEY

The carrier delivering RMC must be timely, flexible, and able to operate within construction site operating constraints [1, 2]. These challenges arise from the value-to-weight ratio of RMC and its highly perishable condition such that it must be discharged from the truck before it hardens [3]. Hence, RMC supply chain management, and especially the DA problem pertaining to support of supply chain process of RMC suppliers in complex and uncertain environments, are crucial issues in the construction industry.

A. Developing an RMC Supply Chain

The literature includes many studies of RMC supply chains in construction projects. Yan et al. (2008) developed an integrated model combining RMC production scheduling and truck dispatching [2]. Park et al. (2011) developed a dynamic simulation model to analyze the RMC supply process by focusing on the tradeoff between the truck mixer dispatching interval and on-site queuing time [1].

Applying DA in RMC supply chain processes involves two important considerations. First, suppliers always focus on productivity issues. One example of time and cost efficiency is minimizing on-site truck mixer idling to reduce operational losses. Meanwhile, the supplier is concerned about maintaining a good reputation. Secondly, contractors as the client representatives are most concerned about timely delivery to ensure no interruptions in concrete placing. However, automated decision tools for supporting RMC supply planning from the supplier perspective such as decisions tools for maintaining service quality are rarely reported in the literature [1].

B. Uncertainty Information in Decision Making

The ME-MCDA approach usually aims to achieve a group consensus on the relative importance of different criteria via discussion and negotiation between the members. However, the weakness of such a decision scheme is that powerful group members may dominate the final outcome, particularly in decision groups consisting of experts from different hierarchical levels [4]. The decision process is most valuable when group members can retrospectively appreciate the differences and similarities of their judgments [5].

Ultimately, the often subjective, ambiguous, and imprecise processes of assigning the criteria weight values (CWs) can cause uncertainty. Thus, subjectivity automatically affects the outcomes of the decision analysis. However, this issue is largely disregarded or inadequately assessed when applying ME-MCDA to solve
construction management problems. Hence, the individual preference ranking, which is a crucial aspect ignored by almost all previous DA studies, must be considered as a part of the group DA in order to obtain objective and reliable multi-expert decisions.

C. Pertinent Decision Aid Support Tools

The Superiority and Inferiority Ranking (SIR) approach developed by Xu (2001) is one of the most effective MCDA methods [6]. The SIR, which is actually a family of methods rather than a single method, is superior to other MCDA methods, (i.e., AHP, ELECTRE III, fuzzy evaluation system and PROMETHEE) [6, 7]. Diverse applications and validation of the method have demonstrated its robustness for solving multi DMs [6-9]. Although SIR is considered a powerful MCDA method, it has some recognized deficiencies. Firstly, no studies have proposed methods of weighting criteria judged by different experts. Using the average weight value as the only criterion excludes considerable information and produces unreliable final results, especially when the relations between attributes and DMs utilities are non-linear [10]. Secondly, the conventional method lacks probability point assessment and explanation with respect to uncertainty judgments by DMs. As described above, this study revised the SIR method by using Monte Carlo simulation (MCS) to obtain the probabilistic pair-wise comparison matrix.

III. RESEARCH METHODOLOGY

A. Building Blocks of Ranking Matrix Flow

The methodology assumes that all criteria must be maximized. Let \( A_1, A_2, \ldots, A_n \) be \( m \) alternatives and \( g_1, g_2, \ldots, g_n \) be \( n \) cardinal criteria and let \( g_i(A) \) be the criteria value (performance) of the \( i \)th alternative \( A \) with respect to the \( j \)th criterion \( g_j \), where \( g_i() \) is a real-valued function \( (i=1,2,\ldots,m; j=1,2,\ldots,n) \). These criteria values form a decision matrix \( D \):

\[
D = \begin{bmatrix}
g_1(A_1) & g_2(A_1) & \cdots & g_n(A_1) \\
g_1(A_2) & g_2(A_2) & \cdots & g_n(A_2) \\
\vdots & \vdots & \ddots & \vdots \\
g_1(A_m) & g_2(A_m) & \cdots & g_n(A_m)
\end{bmatrix}
\]

The generalized criterion is then calculated using the elements of the decision matrix. Performance level can be measured as indices, numerical constants or values in different units for different criteria [9]. The differences between criteria values are used to estimate the intensity of the preference for \( A \) over \( A' \) (Eq. 1):

\[
P(A, A') = f(d) = f \left( g(A) - g(A') \right) \quad (1)
\]

where \( P(A, A') \) is the intensity of the preference for \( A \) over \( A' \) and is a non-decreasing function from \( \mathbb{R} \) (the real number) to \([0,1] \) such that \( f(d) = 0 \) for \( d \leq 0 \) where \( g(A) \leq g(A') \).

The decision maker selects the generalized criteria along with its associated parameters according to the attitude towards the preference structure and intensity of preference. In this study, the Gaussian type is which is the typical generalization selected by users for practical applications, is designated the criterion generalization [6, 11]. The Gaussian type is derived by Eq. 2.

\[
f(d) = \begin{cases} 1 - \exp \left( -\frac{d^2}{2\sigma^2} \right) & \text{if } d > 0 \\ 0 & \text{if } d < 0 \end{cases} \quad (2)
\]

where \( d \) corresponds to the DM perspective and where \( \sigma \) represents the experience obtained with the Normal distribution in statistics [11]. This study assumes that \( d \) and \( \sigma \) are the average value and the standard deviation, respectively, for the criteria for each alternative. For each alternative \( A \), the superiority index \( S_i(A) \) and inferiority index \( I_i(A) \) with respect to the \( j \)th criterion are calculated as follows:

\[
S_i(A) = \sum_{k=1}^{m} P(A, A_k) = \sum_{k=1}^{m} f_i \left( g_j(A) - g_j(A_k) \right) \quad (3)
\]

\[
I_i(A) = \sum_{k=1}^{m} P(A, A_k) = \sum_{k=1}^{m} f_i \left( g_j(A) - g_j(A_k) \right) \quad (4)
\]

where if \( d \leq 0, f_i(d) = 0 \), then \( S_i(A) \) and \( I_i(A) \) are the superiority and inferiority scores of alternative \( A \), respectively.

\[
S = \begin{bmatrix}
S_1(A_1) & \cdots & S_1(A_m) & \cdots & S_1(A_n) \\
S_2(A_1) & \cdots & S_2(A_m) & \cdots & S_2(A_n) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
S_n(A_1) & \cdots & S_n(A_m) & \cdots & S_n(A_n)
\end{bmatrix}
\]

\[
I = \begin{bmatrix}
I_1(A_1) & \cdots & I_1(A_m) & \cdots & I_1(A_n) \\
I_2(A_1) & \cdots & I_2(A_m) & \cdots & I_2(A_n) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
I_n(A_1) & \cdots & I_n(A_m) & \cdots & I_n(A_n)
\end{bmatrix}
\]

Equations 3 and 4 determine the total intensity of the superiority and inferiority indexes, respectively, for alternative \( A \) in comparison with other alternatives. The S-matrix indicates the intensity of superiority of each alternative for each criterion whereas I-matrix generates information about the intensity of inferiority. The S- matrix and I-matrix forms are shown above.

Matrices S and I provide better information compared to the original matrix D because it considers the intensity of superiority and inferiority given by the generalized criteria.
B. Aggregating Superiority and Inferiority Scores

In this study, grey system theory is used to deal with superiority and inferiority scores and with criteria weightings defined by DMs to formulate the S-flow $\varphi^s(\cdot)$ and the I-flow $\varphi^i(\cdot)$. The process is described below [9].

\[
F^s_i = (\max_i S_i(A) \cdots \max_i S_i(A) \cdots \max_i S_i(A))
\]

\[
F^i_i = \left[ f^i_1 \cdots f^i_n \cdots f^i_n \right]
\]

\[
D_i = \left[ \begin{array}{ccc}
     f^i_1 & \cdots & f^i_n \\
     \vdots & \ddots & \vdots \\
     f^i_n & \cdots & f^i_n
\end{array} \right]
\]

where $F^s_i$ is the reference score matrix, $f^i$ is the index score and $D_i$ is the combination matrix between $F^i_i$ and $S^i_i$-matrix and $I^i_i$-matrix sets provide further explanation where the $S^i_i$-distinguishing coefficient, which has a value of 0.5; $\omega$ is the $\varphi$<sub>DC</sub> which is denoted by $DC_i(\varphi^i(A))$ and $DC_i(\varphi^s(A))$ respectively. Linear normalization of the matrix gets:

\[
DC_i = \left[ \begin{array}{ccc}
     C^i_1 & \cdots & C^i_n \\
     \vdots & \ddots & \vdots \\
     C^i_n & \cdots & C^i_n
\end{array} \right]
\]

Derivation of grey relational grade gets:

\[
e^{\text{rel}}(j) = \min \left[ \frac{\min C^i_j - C^s_j}{C^s_j - C^s_j} + \rho \frac{\max C^i_j - C^s_j}{C^s_j - C^s_j} \right]
\]

where $C^i_j$ and $C^s_j$ are the representation of $f^i_j$ and $f^s_j$, respectively, after normalization in the $D_i$ matrix set, which is denoted by $DC_i$, $\rho \in [0,1]$; $\rho$ indicates the distinguishing coefficient, which has a value of 0.5; $e^{\text{rel}}(j)$ denotes the grey relations grade.

\[
\varphi^i(A) = \sum_{j=1}^n \omega_j e^{\text{rel}}(j), \quad \sum_{j=1}^n \omega_j = 1 \quad (\omega_j \geq 0)
\]

where $\omega_j$ is the $j$th CW value. Notably, $\rho = 0.5$ is the optimal value for the real-world problem. If $\rho = 1$, all information is unknown; if $\rho = 0$, all information is known. Normally, however, only part of the information is known in the real field [9].

C. Populate AHP Prioritization Matrix Inherent Uncertainties

The criteria weight (CW) can be estimated by either DMs or by the responses to questionnaire surveys of project stakeholders. Even so, uncertainty occurs when DMs apply their individual personal contention structures in the CW evaluation. Due to its popularity, this study implements AHP prioritization method to estimate and determine the weight value for each criterion. However, AHP is deterministic process that lacks probability point estimation.

Based on the capability, applicability and superiority of the method considering the uncertainty of information, this study applies the MCS and virtual-scale prioritization matrix method within AHP process.

C.1. Virtual-scale Criteria Weight Values

Let $a$ represents the element value on the actual AHP scale; that is, $a = \frac{1}{\rho}$ through $a = 9$, and $v$ indicates the corresponding value on the virtual scale. The conversion of the prioritization matrix elements to a virtual scale is described in Eqs. 9 and 10 [12].

\[
v(a) = \begin{cases} 
\frac{2a - 1}{a} & \text{if } a < 1 \\
\frac{a}{a} & \text{if } a \geq 1 \\
\frac{2v - 1}{v} & \text{if } v < 1 \\
v & \text{if } v \geq 1 
\end{cases}
\]

The next part assigns uncertainties. In this study, AHP prioritization matrix ratings on the virtual scale are assumedly distributed as triangular random variables. Additionally, using triangular distributions for early project planning applications is well supported in the literature [12, 13]. Thus, the probabilistic prioritization matrix based on the virtual-scale is shown below.

\[
X_i = \begin{pmatrix}
X_{i1} & X_{i2} & \cdots & X_{in} \\
X_{i2} & X_{i3} & \cdots & X_{in} \\
\vdots & \vdots & \ddots & \vdots \\
X_{in} & X_{i1} & \cdots & X_{i2}
\end{pmatrix}
\]

\[
X_i = \text{Triangular}(u_i, v_i, y_i) \quad \text{with } X_i \text{ being the } i\text{th row of the matrix.}
\]

\[
X_i = \begin{pmatrix}
2 - X_{i1} & 1 & \cdots & X_{in} \\
2 - X_{i2} & 1 & \cdots & X_{in} \\
\vdots & \vdots & \ddots & \vdots \\
2 - X_{in} & 1 & \cdots & X_{i1} \\
2 - X_{i1} & 1 & \cdots & X_{in}
\end{pmatrix}
\]

\[
X_i = \begin{pmatrix}
2 - X_{i1} & 2 - X_{i2} & \cdots & 2 - X_{in} \end{pmatrix}
\]

where $(u_i, v_i, y_i)$ represents the minimum, most likely and maximum virtual-scale values of the triangular distribution within the probabilistic prioritization matrix.
C.2. Multi-expert Judgments Representation

The first step in the MCS is selecting an AHP prioritization matrix by sampling from the random matrix (probabilistic prioritization matrix). The lower-left elements are then computed using Eq. 12, and the resulting sample matrix is converted to the actual AHP scale using Eq. 10. The simulations in this work are repeated 500 times. The prioritization matrix is then calculated to obtain the CWs after the standard AHP processes obtains the synthesized matrix and combs out the performance consistency. This study also considers the consistency of DM performance to obtain rigorous results for the final decision outcome (CR≥0.1).

D. Flow-based Ranking

The ranking process follows a procedure similar to the one above. The ranking is based on the I-flow. S-flow \( \phi'(A) \) is calculated using Eq. 5. Likewise, the I-flow \( \phi'(A) \) is calculated by changing Eq. 5 from “max” to “min” following the same calculation procedure of the S-flow \( \phi'(A) \). These values are then used to rank the alternatives. Brans et al. (1986) and Xu (2001) simplified the problem by using synthesizing flows such as the complete pre-order net flow (n-flow) and relative flow (r-flow) ranking [6, 11] when a complete ranking is requested by the DMs [6]. The \( \phi(A) \) can obtain a complete ranking \( \Re \) of the alternatives. The complete ranking pre-order n-flow equation:

\[
\phi_n(A) = \phi'(A) - \phi'(A)
\]

(13)

where \( \phi'(A) \) and \( \phi'(A) \) are S-flow and I-flow respectively.

E. Global Ranking Analysis

The first phase is the filtering phase, which ranks alternatives as high or low where 1/4 or 25% of the alternatives receive a high ranking and the remaining 3/4 or 75% receive a low ranking. The element that indicates the position of the upper quartile is given by the following expression:

\[
x = n / 4 \text{ (round up)}
\]

(14)

where \( n \) is the total number of ranked alternatives. Similarly, the exploration is performed by an inverse analysis where 25% of the alternatives receive a low ranking and the remaining 75% receive a high ranking. The element indicating the position of the lower quartile is given by the following expression:

\[
y = 3n / 4 + 1 \text{ (truncate)}
\]

(15)

where \( n \) is the total number of ranked alternatives. In this case, the number of points is computed as follows: 1 for the last position (the upper quartile limit: \( x \)), 2 for the next to last position, ..., \( x \) for the first position. The points gained by each alternative are totaled, and the alternatives receive a score called the strength of the alternative, \( F_i \), which is given by

\[
F_i = \sum_{k=1}^{n} \left( x - j + 1 \right) q_{kj} \quad \forall i, k \quad \forall j = 1, ..., x
\]

(16)

where

\[
q_{kj} = \begin{cases} 1, & \text{if the alternative } i \text{ is in the position } j \text{ for the decision maker } k \\ 0, & \text{otherwise} \end{cases}
\]

\( i \) corresponds to the alternatives in the upper quartile \( (i = 1, 2, 3, ..., m) \); \( j \) is the position in the upper quartile, which ranges from the 1st position to the upper quartile limit \( (x) \) \( (j = 1, 2, 3, ..., x) \) and, \( k \) represents the decision maker \( (k = 1, 2, ..., n) \).

Similarly, the lower positional count is made, where a number of points are the inverse attributes of each position of the evaluation, because higher points should be considered for the alternatives in the worst position [14], which ensures that the weakest alternatives have the largest point accumulations. The points gained by each alternative are totaled and the alternatives receive a score \( f_j \), which is the weakness of the alternative given by

\[
f_j = \sum_{i=1}^{n} \sum_{j=1}^{m} \left( j - y + 1 \right) q_{ij} \quad \forall i, k \quad \forall j = y, ..., m
\]

(17)

where

\[
q_{ij} = \begin{cases} 1, & \text{if the alternative } i \text{ is in the position } j \text{ for the decision maker } k \\ 0, & \text{otherwise} \end{cases}
\]

\( i \) corresponds to the alternatives in the lower quartile \( (i = 1, 2, 3, ..., m) \); \( j \) is the position in the lower quartile, which varies from the 1st position of the lower quartile limit \( (y) \) \( (j = y, 1, 2, ..., y) \), and \( k \) represents a decision maker \( (k = 1, 2, ..., n) \).

Further, the second phase is called Veto phase. The positional count considers strength \( (F_j) \) and the weakness \( (f_j) \) of the alternatives. This phase reveals the relation between the strength and weakness of the alternatives \( (F_j > f_j) \). This phase can detect very high opposition to the alternatives and can use a veto function to eliminate the alternatives classified as worst by the majority of DMs.

The last part is the Choosing phase which calculates \( \alpha_i = F_i - f_i \), which is the strength of the intensity of preference for alternatives. The alternative with the largest number of points \( (i.e., \text{the highest } \alpha_i) \) can then be selected.

IV. CASE STUDY: RMC UNLOADING TYPE
The case data are excerpted from previous research in RMC supply chain conducted by Park et al. (2011) [1]. The nine mixer truck performance criteria are \(g_i\) to \(g_9\), which are described as follows: (a) Average truck mixer dispatching interval; (b) Average time to stop; (c) Average time to load mix; (d) Average time to slump test; (e) Average time to haul; (f) Average on-site queuing time; (g) Average time to unload; (h) Average time to return; (i) Average time for delivery.

Tables 1 and 2 show the criteria and their preference structures. Two case scenarios are considered in this numerical example; Scen_1 assigns deterministic weights to the selected criteria considered in the single DA, and Scen_2 assigns stochastic weights to the selected criteria considered in the multi-expert DA.

### A. S-matrix and I-matrix Workflow

Eqs. 1-4 obtain the following resulting decision matrix (D), S-matrix (S), and I-matrix (I):

#### TABLE 1

<table>
<thead>
<tr>
<th>Types of unloading</th>
<th>Avg truck mixer dispatching interval</th>
<th>Avg time to position</th>
<th>Avg time to load mix</th>
<th>Avg time for slump test</th>
<th>Avg time to haul</th>
<th>Avg queuing time on-site</th>
<th>Avg time to unload</th>
<th>Avg time to return</th>
<th>Avg time taken for delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crane and Skip</td>
<td>18.3</td>
<td>1.58</td>
<td>1.27</td>
<td>0.61</td>
<td>23.04</td>
<td>29.94</td>
<td>20.05</td>
<td>24.92</td>
<td>103.81</td>
</tr>
<tr>
<td>Wheel Barrow</td>
<td>20.7</td>
<td>0.79</td>
<td>1.05</td>
<td>1.05</td>
<td>21.50</td>
<td>22.27</td>
<td>25.90</td>
<td>20.50</td>
<td>93.59</td>
</tr>
<tr>
<td>Tremie Pour</td>
<td>6.30</td>
<td>1.84</td>
<td>1.55</td>
<td>2.95</td>
<td>22.31</td>
<td>12.01</td>
<td>9.50</td>
<td>24.44</td>
<td>74.60</td>
</tr>
<tr>
<td>Pump Pour</td>
<td>8.10</td>
<td>2.11</td>
<td>1.04</td>
<td>2.76</td>
<td>21.33</td>
<td>16.96</td>
<td>13.85</td>
<td>21.67</td>
<td>79.72</td>
</tr>
<tr>
<td>Direct Pour</td>
<td>7.50</td>
<td>1.50</td>
<td>1.33</td>
<td>3.07</td>
<td>22.21</td>
<td>12.06</td>
<td>9.24</td>
<td>24.67</td>
<td>74.08</td>
</tr>
<tr>
<td>All Types(^*)</td>
<td>13.15</td>
<td>1.55</td>
<td>1.23</td>
<td>2.81</td>
<td>22.38</td>
<td>20.76</td>
<td>15.91</td>
<td>23.96</td>
<td>88.60</td>
</tr>
</tbody>
</table>

#### TABLE 2

<table>
<thead>
<tr>
<th>Types of criteria</th>
<th>Avg truck mixer dispatching interval</th>
<th>Avg time to position</th>
<th>Avg time to load mix</th>
<th>Avg time for slump test</th>
<th>Avg time to haul</th>
<th>Avg queuing time on-site</th>
<th>Avg time to unload</th>
<th>Avg time to return</th>
<th>Avg time taken for delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferred limit</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
</tr>
<tr>
<td>Type of criterion</td>
<td>Min</td>
<td>(\sigma = 6.07)</td>
<td>(\sigma = 0.44)</td>
<td>(\sigma = 0.31)</td>
<td>Min</td>
<td>(\sigma = 0.39)</td>
<td>Min</td>
<td>Min</td>
<td>Min</td>
</tr>
<tr>
<td>Gaussian Criterian</td>
<td>(\sigma = 6.39)</td>
<td>(\sigma = 0.63)</td>
<td>(\sigma = 6.90)</td>
<td>(\sigma = 6.43)</td>
<td>(\sigma = 1.83)</td>
<td>(\sigma = 11.77)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIR-Grey</td>
<td>Weight (Scen_1)</td>
<td>0.113</td>
<td>0.046</td>
<td>0.050</td>
<td>0.125</td>
<td>0.118</td>
<td>0.375</td>
<td>0.087</td>
<td>0.056</td>
</tr>
</tbody>
</table>

#### TABLE 3

<table>
<thead>
<tr>
<th>Types of criteria</th>
<th>Avg truck mixer dispatching Interval</th>
<th>Avg time to Position</th>
<th>Avg time to load mix</th>
<th>Avg time for slump test</th>
<th>Avg time to haul</th>
<th>Avg queuing time on-site</th>
<th>Avg time to unload</th>
<th>Avg time to return</th>
<th>Avg time taken for delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg truck mixer dispatching interval</td>
<td>1</td>
<td>2-(0,3,5)</td>
<td>2-(1,3,5)</td>
<td>2-(0,2,5)</td>
<td>2-(0,2,4)</td>
<td>2-(2,4,6)</td>
<td>2-(1,3,5)</td>
<td>2-(3,5,8)</td>
<td>2-(2,1,2)</td>
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<tr>
<td>Avg time to position</td>
<td>(0,3,5)</td>
<td>1</td>
<td>2-(1,1,2)</td>
<td>2-(2,1,5)</td>
<td>2-(1,2)</td>
<td>2-(0,1,3)</td>
<td>2-(0,1,5)</td>
<td>2-(1,1,3)</td>
<td>2-(6,4,1)</td>
</tr>
<tr>
<td>Avg time to load mix</td>
<td>(1,3,5)</td>
<td>(1,-1,2)</td>
<td>1</td>
<td>2-(1,1,2)</td>
<td>2-(2,4,2)</td>
<td>2-(1,3,4)</td>
<td>2-(1,2)</td>
<td>2-(1,2)</td>
<td>2-(6,5,0)</td>
</tr>
<tr>
<td>Avg time for slump test</td>
<td>(0,3,5)</td>
<td>(1,-1,2)</td>
<td>(1,1,2)</td>
<td>1</td>
<td>2-(1,1,3)</td>
<td>2-(0,1,3)</td>
<td>2-(1,3)</td>
<td>2-(1,3)</td>
<td>2-(6,4,1)</td>
</tr>
<tr>
<td>Avg time to haul</td>
<td>(0,2,4)</td>
<td>(1,-1,3)</td>
<td>(4,0,3)</td>
<td>(1,1,3)</td>
<td>1</td>
<td>2-(0,2,4)</td>
<td>2-(0,1,3)</td>
<td>2-(1,2)</td>
<td>2-(6,4,0)</td>
</tr>
<tr>
<td>Avg queuing time on-site</td>
<td>(2,4,1)</td>
<td>(0,1,0)</td>
<td>(0,1,3)</td>
<td>(1,0,1)</td>
<td>1</td>
<td>2-(1,1,4)</td>
<td>2-(1,3)</td>
<td>2-(1,3)</td>
<td>2-(6,4,0)</td>
</tr>
<tr>
<td>Avg time to unload</td>
<td>(1,3,5)</td>
<td>(0,1,5)</td>
<td>(3,0,3)</td>
<td>(2,1,2)</td>
<td>(1,1,4)</td>
<td>1</td>
<td>2-(1,0,3)</td>
<td>2-(1,3)</td>
<td>2-(6,5,1)</td>
</tr>
<tr>
<td>Avg time to return</td>
<td>(3,5,8)</td>
<td>(1,-1,3)</td>
<td>(1,-1,3)</td>
<td>(1,2,4)</td>
<td>(0,1,3)</td>
<td>1</td>
<td>2-(6,5,1)</td>
<td>2-(1,3)</td>
<td>2-(6,5,1)</td>
</tr>
<tr>
<td>Avg time taken for delivery</td>
<td>(-2,1,2)</td>
<td>(-6,-4,1)</td>
<td>(-6,5,0)</td>
<td>(-5,4,0)</td>
<td>(-4,2,0)</td>
<td>(-7,-4,3)</td>
<td>(-5,-3,1)</td>
<td>(-6,-5,1)</td>
<td>1</td>
</tr>
</tbody>
</table>

\(^*\) means (minimum, mode, maximum) preference value after applying virtual-scale rules.
After calculating the $S$-flow and $I$-flow matrices based on the grey relational grade aggregation method, the $S$-flow and $I$-flow matrices denoted by $S\epsilon^c_i(j)$ and $I\epsilon^c_i(j)$ respectively, are as follows:

\[
S\epsilon^c_i(j) = 
\begin{bmatrix}
0.73 & 0.41 & 0.46 & 0.85 & 1.00 & 1.00 & 0.52 & 1.00 & 1.00 \\
1.00 & 0.33 & 0.33 & 0.34 & 0.48 & 1.00 & 0.33 & 0.53 \\
0.33 & 0.52 & 1.00 & 0.76 & 0.44 & 0.33 & 0.33 & 0.80 & 0.33 \\
0.34 & 1.00 & 0.39 & 0.66 & 0.33 & 0.36 & 0.36 & 0.39 & 0.35 \\
0.33 & 0.40 & 0.50 & 1.00 & 0.42 & 0.33 & 0.33 & 0.88 & 0.33 \\
0.43 & 0.41 & 0.45 & 0.67 & 0.46 & 0.43 & 0.39 & 0.69 & 0.44 \\
\end{bmatrix}
\]

\[
I\epsilon^c_i(j) = 
\begin{bmatrix}
0.94 & 0.76 & 0.86 & 1.00 & 1.00 & 0.78 & 1.00 & 1.00 & 1.00 \\
1.00 & 0.33 & 0.33 & 0.33 & 0.37 & 0.37 & 1.00 & 0.33 & 0.79 \\
0.33 & 0.93 & 1.00 & 0.99 & 0.76 & 0.76 & 0.34 & 0.98 & 0.34 \\
0.38 & 1.00 & 0.59 & 0.90 & 0.33 & 0.33 & 0.49 & 0.41 & 0.42 \\
0.36 & 0.70 & 0.91 & 1.00 & 0.71 & 0.71 & 0.33 & 1.00 & 0.33 \\
0.58 & 0.74 & 0.82 & 0.93 & 0.78 & 0.78 & 0.57 & 0.89 & 0.64 \\
\end{bmatrix}
\]

Meanwhile, $S$-flow $\phi^c_i$ and likewise, $I$-flow $\phi^c_i$ is obtained by changing Eq. 5 from “max” to “min” and then following the same calculation procedure as that for $S$-flow $\phi_i$.

D. Individual Preference Ranking Analysis

The first phase is analyzing upper positional count $(F_1)$, which is the strength of the alternative, and lower positional count $(F_2)$ count, which is the weakness of the alternative. Where the number of alternatives analyzed in the demonstration case is $n = 6$

- Element of location-upper quartile: $x = n/4 = 6/4 = 1/5 = 1$
- Element of location-lower quartile: $y = 3n/4 + 1 = 3 \times 6/4 + 1 = 5.5 = 5$

The second phase performs a positional count of the alternatives, based on the Strength $(F_1)$ and the Weakness $(F_2)$ of the alternatives result. The third phase focuses on the Strength of the intensity of preference for the alternatives based on $\sigma_i$. The comparison shows that $A_5$ has the best performance among the remaining alternatives as $\alpha_5 = 345$. Table 4 shows the detailed individual results, including the performance rankings of the remaining alternatives.

E. Discussion

The method proposed in this study obtained satisfactory decision results. The alternative chosen, Wheel Barrow of alternative 2 ($A_2$) is placed at the top of major individual rankings followed by Pump Pour ($A_4$), Direct Pour ($A_3$), Tremie Pour ($A_1$), Crane and Skip ($A_5$), and All types ($A_6$). Further, analyses of individual preferences using RS-SIR method indicate that $A_2$ received ultimate decision position in first rank.

The global individual preference results above confirm that Wheel Barrow is the best selection. However, when using conventional SIR, the results show a different order compared to the RS-SIR, i.e., alternative 5 ($A_5$) is superior to other alternatives. As stated in Table 4, alternatives $A_2$, $A_4$ and $A_5$ were not eliminated. Nevertheless, the strength of the intensity was lower in all three alternatives than in $A_2$. Alternative $A_5$ which is apparently as attractive as alternative $A_5$ was also ranked.

**TABLE 4**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Upper positional count $(F_1)$</th>
<th>Lower positional count $(F_2)$</th>
<th>$f_i$</th>
<th>$\alpha_i$</th>
<th>$f_i \geq F_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>345</td>
<td>0</td>
<td>345</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>345</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>345</td>
<td>345</td>
<td>6</td>
</tr>
</tbody>
</table>

**Fig. 1. Sequential ranking.**

The global individual preference results above confirm that Wheel Barrow is the best selection. However, when using conventional SIR, the results show a different order compared to the RS-SIR, i.e., alternative 5 ($A_5$) is superior to other alternatives. As stated in Table 4, alternatives $A_2$, $A_4$ and $A_5$ were not eliminated. Nevertheless, the strength of the intensity was lower in all three alternatives than in $A_2$. Alternative $A_5$ which is apparently as attractive as alternative $A_5$ was also ranked.
third and fourth by multiple DMs respectively. However, $A_3$ and $A_4$ were excluded from positional counting due to their reduced relevance when they were positioned outside both quartiles for all other DMs.

This case analysis shows that the aggregation procedure considers all rankings instead of only the highest ranking. Also, the Veto step accounts for alternatives rejected by the group. For instance, $A_5$ and $A_6$ were eliminated due to strong opposition to both alternatives (Table 4). All DMs ranked $A_6$ lowest but ranked $A_1$ highest whereas $A_6$ and $A_1$ are ranked equally where $f_i \geq F_i$. Finally, the analysis of global result indicates that the aggregation of individual results is consistent with individual preferences of DMs.

V. CONCLUSION

As the complexity of decision processes increase, the uncertainty of evaluations increases. Hence, uncertainty is an important consideration in ME-MCDA. Moreover, this study integrates AHP and MCS to account for uncertain information in the DA process. The proposed method considers the perspective of multiple experts within the conventional SIR method. Additionally, applying the virtual-scale method in the AHP process achieves consistent stochastic DMs judgments.

The method can also solve complex problems in which DMs assign ranges of uncertainty in a prioritization matrix rating. The outputs generated from the example reveal that the rankings of alternatives depend on the weights assigned in the criteria and aggregation procedure. Therefore, an appropriate aggregation procedure enables DMs to rank alternatives reliably and to capture the characteristics of the problem. The lack of guidance for setting the values of preference and indifference thresholds may sometimes invalidate the overall ranking process. Hence, a good understanding of DM criteria is needed when setting these thresholds.

REFERENCES